2/7/18 "An eye for an eye only ends up making the whole world blind."-Gandhi

HW: "Optimization packet" page 126 #5
Test 1 on Thursday 2/15

AIM: How do we minimize materials used?

Warm Up:

(can)

1) What is the formula for the surface area of a right cylinder?

$SA = 2\pi r^2 + 2\pi r h$

2) What is the formula for the volume of a right cylinder?

$V = \pi r^2 h$

HW Check:

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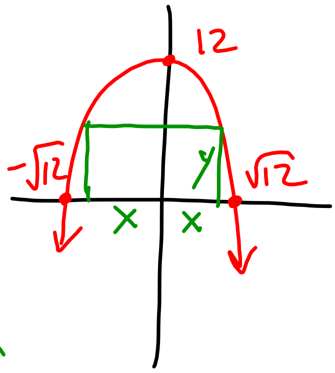
1)



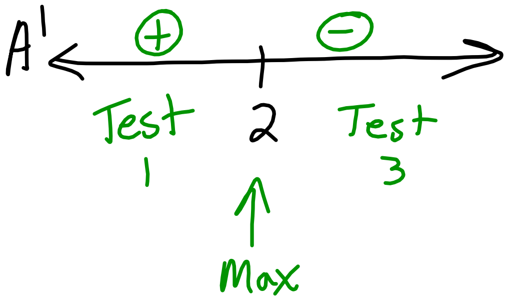
$y = 12 - x^2$
$0 = 12 - x^2$
$x^2 = 12$
$x = \pm\sqrt{12}$

Rest.
$0 \le y \le 12$
$0 \le x \le \sqrt{12}$

Primary:
$A = 2xy$

Secondary:
$y = 12 - x^2$

$A = 2x(12 - x^2)$
$A = 24x - 2x^3$



$A' = 24 - 6x^2$
$0 = 24 - 6x^2$
$6x^2 = 24$
$x^2 = 4$
$x = \pm 2$ reject -2

Find y:
$y = 12 - 2^2$
$y = 12 - 4$
$y = 8$

Max Area
$A = 2(2)(8)$
$\boxed{A = 32 \text{ units}^2}$

Page 124 rest: $r > 0$ $h > 0$

Secondary

4. What is the radius of a cylindrical soda can with volume of 512 cubic inches that will use the minimum material?

Surface Area

$SA = 2\pi r^2 + 2\pi r h$

$V = 512$

$\frac{512}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$

$h = \frac{512}{\pi r^2}$

$SA = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{512}{\cancel{\pi} r^{\cancel{2}}}\right)$

$SA = 2\pi r^2 + 2\left(\frac{512}{r}\right)$

$SA = 2\pi r^2 + 1024 r^{-1}$

$SA' = 4\pi r - 1024 r^{-2}$

$r^2\left(0 = 4\pi r - \frac{1024}{r^2}\right)$

Undefined when $r = 0$ but the restriction tells us $r > 0$.

$0 = 4\pi r^3 - 1024$

$1024 = 4\pi r^3$

$\frac{1024}{4\pi} = r^3 \longrightarrow \frac{256}{\pi} = r^3$

$\sqrt[3]{\frac{256}{\pi}} = r$

≈ 4.335

SA': $\ominus$ | $\oplus$

Test 3 $\sqrt[3]{\frac{256}{\pi}}$ Test 5

min ✓

Radius is

$\sqrt[3]{\frac{256}{\pi}}$ inches

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Example 3) A real estate company owns 100 apartments in New York City. At $1,000 per month, each apartment can be rented. However, for each $50 increase, there will be two additional vacancies. How much should the real estate company charge for rent to maximize its revenues?

$50 increases	Rent	Apts. ~~rented~~	Revenue
0	1000	100	100000
1	1050	98	102900
2	1100	96	105600
3			
4			
50	3500	0	0
x			

Let x = # of increases

$$0 \le x \le 50$$

Revenue = Rent × Appts

$$R = (1000 + 50x)(100 - 2x)$$

$$R = 100000 - 2000x + 5000x - 100x^2$$

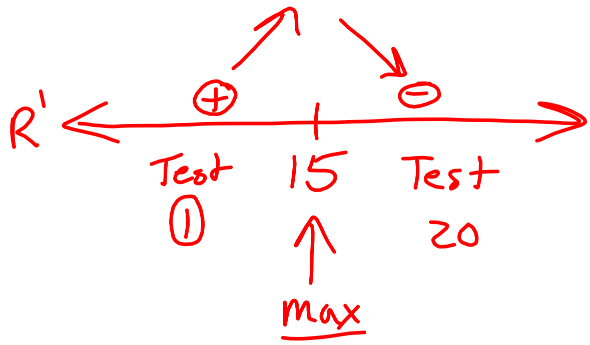
$$R = -100x^2 + 3000x + 100000$$

$$R' = -200x + 3000$$

$$0 = -200x + 3000$$

$$\frac{200x}{200} = \frac{3000}{200}$$

$$x = 15$$



Amount Rent:

$$= 1000 + 50(x)$$

$$= 1000 + 50(15)$$

$$\boxed{= \$1750}$$

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restrictions: $r>0$ $h>0$

$$\boxed{V=1000} \rightarrow \frac{1000}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$h = \frac{1000}{\pi r^2}$$

Min SA

$$SA = 2\pi r^2 + 2\pi r h$$

$$SA = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$$

$$SA = 2\pi r^2 + \frac{2000}{r}$$

$$SA = 2\pi r^2 + 2000r^{-1}$$

$$SA' = 4\pi r - 2000r^{-2}$$

$$r^2\left(0 = 4\pi r - \frac{2000}{r^2}\right)$$

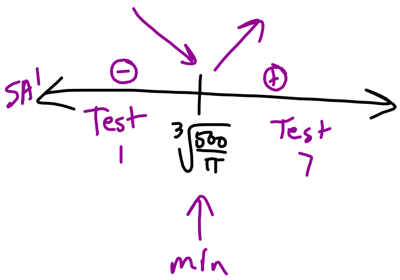
Undefined when $r=0$ but $r \neq 0$ because of restriction $r>0$

$$0 = 4\pi r^3 - 2000$$

$$\frac{2000}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\frac{500}{\pi} = r^3 \qquad r = \sqrt[3]{\frac{500}{\pi}}$$

$$r \approx 5.4$$



Find h:

$$h = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2}$$

$$\boxed{r = \sqrt[3]{\frac{500}{\pi}}\text{ cm} \qquad h = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2}\text{ cm}}$$

(*) Ideal surface area to minimize is to have height = diameter